Modelling Built-up Steam Turbine Rotor Using Finite Difference Method

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Abstract

The low-pressure cylinders in the two 120MW turbines at Verve Energy's Kwinana Power Station house built-up rotors with discs shrink fitted to the shaft. Verve wants to estimate temperatures and stresses in the discs to ensure safe turbine operation. The objective is to produce an in-house application for Verve to determine temperature and stress fields within the disc and shaft for known inputs of geometry, interference fit, material properties, rotational speed and boundary temperature. A model of a single disc shrink fitted onto a shaft will be implemented using the finite difference method in MATLAB, at Verve's request. Stresses from centrifugal loading, thermal gradients, blade pressure loading and shrink fitting will be calculated and combined under the principle of superposition to give the three dimensional stress state at each point in the cylinder-disc assembly. The application will provide Verve with a tool for analysing turbine behaviour and bring previously external work inside the company thereby decreasing Verve's reliance upon external consultants. Key deliverables are a project report, the application and an operation manual for the application.

1. Introduction

1.1 Background Information

A problem with built-up turbines is cracking at the bore of the discs, which can potentially cause catastrophic failure of the turbine. A major turbine failure at Kwinana Power Station could cost Verve $5M to $10M in total for repairs and lost power generation. To reduce the risk of turbine failure a better understanding of the stress field within the disc is needed.

The main motivation for this project is Verve's goal of building up internal engineering expertise. A significant number of new engineers have joined the company recently and this has been used as a turning point to bring some external activities back inside the organisation.

1.2 Current State of the Art

Verve runs an ongoing turbine disc inspection program to detect cracking. Information from these inspections is used in conjunction with other analyses (e.g. finite element analysis or FEA) to decide how long the turbines can be safely run before the next inspection. So far FEA of turbine rotors has been outsourced to third parties and the stresses present are already well understood.

Verve now wants the in house capability to perform this type of analysis. Since Verve does not have licenses for any commercial FEA software, they want to use the finite difference method (FDM) to perform analysis. FDM analysis will be used as the first stage of a two-stage analysis. The results from FDM analysis will be used to decide whether or not a full scale FEA need be performed as a second stage of analysis.

1.3 Objectives

The objective of the project is to create a program that predicts the temperature and stress fields within the turbine disc for inputs of geometry, material properties, rotational speed and boundary temperature. The program will calculate temperature and stress fields using the finite difference method. Knowledge of the stresses also allows prediction of the maximum rotational speed achievable before the shrink fit becomes ineffective.

2. Model Description

A single stage of a built-up turbine rotor consisting of a turbine disc shrink fitted onto a cylindrical shaft will be modelled using the finite difference method. Three sources of rotor stress will be considered in this model:

1. Body forces due to the inertia of the disc and attached turbine blades,
2. Axial and radial temperature variation within the rotor, and
3. Force from the interference fit between disc and shaft.

Equations for stresses from these sources are each calculated separately before being combined under the principle of superposition (Kearton 1945). Nomenclature used in the following sections is as follows:

ϱ	Density	$(r/u/h)_b$	at bore of cylinder
ν	Poisson's Ratio	$(r/u/h)_i$	at cylinder-disc interface
ω	Angular velocity	$(r/u/h)_r$	at rim of disc
E	Young's Modulus	σ_r	Radial stress
a	Finite difference grid spacing	σ_t	Tangential stress
l	Length of cylinder section	σ_z	Axial stress
r	Radius	ε_r	Radial strain
u	Radial displacement	ε_r	Tangential strain
h	Width of disc (function of radius)	ε_r	Axial strain

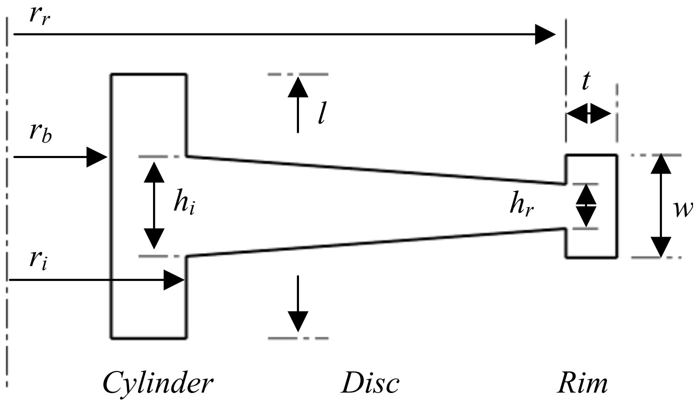

Figure 1 Cross section of rotor showing input dimensions.

	Variable name	Value	[unit]	Description
Rotor dimensions	N	50	[1]	Number of grid points in disc
	rb	0.127	[m]	Radius at bore of shaft
	ri	0.165	[m]	Radius at interface between cylinder and disc
	rr	0.356	[m]	Radius at rim of disc
	l	0.095	[m]	Length of cylinder stage associated with disc
	hi	0.045	[m]	Disc width at interface
	hr	0.017	[m]	Disc width at rim
	t	0.034	[m]	Rim thickness
	w	0.054	[m]	Rim width
Other constants	E	2.07E+11	[Pa]	Young's Modulus
	rho	7833.440	[kg/m^3]	Density
	omega	314.159	[1/s]	Angular velocity
	nu	0.300	[1]	Poisson's Ratio
	nB	200.000	[1]	Number of blades
	mB	0.333	[kg]	Mass per blade
	eps	0.000	[1]	Finite difference tolerance

Table 1 Name and description of user inputs to the program entered via Microsoft Excel spreadsheet.

Figure 1 shows a cross section of the rotor with dimensions labelled. Table 1 is the Microsoft Excel spreadsheet used for input. Rotor dimensions and constants such as rotational speed and material properties are entered in the shaded cells. The dimensions in Table 1 are arbitrary inputs used for demonstration.

2.1 Stress from Inertia of Disc and Attached Blades

Because different assumptions are made for each component, the equations for disc and cylinder cannot be solved directly and instead an iterative approach is used. The disc is modelled as a thin disc of variable thickness in plain stress whereas the cylinder is taken to be a thick cylinder under plain strain conditions. The two sections are linked together by the internal force acting on the disc from the cylinder and the equal and opposite internal force from the cylinder on the disc. The following equations are all given in or derived from equations found in Kearton (1945) and Timoshenko & Goodier (1951).

Radial displacement at the outer diameter (OD) of the cylinder is found using equation (1) which gives displacement for thick cylinders subject to rotation only. This first step assumes the radial stress at the interface is zero (P = 0). Displacements are then calculated through the disc using equations $(2)-(2c)$ and an initial blade load, F_1, is calculated using equations $(3a)$ and $(3b)$, where $(\sigma_r)_{N+1}$ is the radial stress at the rim of the disc.

$$u = \frac{\rho\omega^2 r}{8\mathrm{E}}\left\{\frac{3-5\nu}{1-\nu}\left(r_b^2+r_i^2\right)+\frac{(1+\nu)(3-2\nu)}{1-\nu}\cdot\frac{r_b^2 r_i^2}{r^2}-\frac{(1+\nu)(1-2\nu)}{1-\nu}\cdot r^2\right\} \qquad (1)$$

$$u_{n+1} = \frac{1}{1+\frac{a\cdot A_n}{2}}\left\{\left(2-\frac{a^2\cdot B_n}{r_n}\right)\cdot u_n+\left(\frac{a\cdot A_n}{2}-1\right)\cdot u_{n-1}+a^2\cdot C_n\right\} \qquad (2)$$

$$A_n = \frac{h_{n+1} - h_{n-1}}{2ah_n} + \frac{1}{r_n} \tag{2a}$$

$$B_n = \nu \frac{h_{n+1} - h_{n-1}}{2ah_n} - \frac{1}{r_n} \tag{2b}$$

$$C_n = -\frac{1-\nu^2}{\mathrm{E}} \rho\omega^2 r_n \tag{2c}$$

$$Q = \frac{t \cdot w}{r_{N+1}^2} \mathrm{E} \cdot u_{N+1} - \rho\omega^2 r_{N+1}^3 \tag{3a}$$

$$F_1 = \frac{2\pi}{n_B} \left\{ r_{N+1} \cdot Q + (\sigma_r)_{N+1} \cdot r_{N+1} \cdot h_{N+1} \right\} \tag{3b}$$

Next, displacements are calculated for a case with unit radial stress ($P = 1$) at cylinder OD to determine a correcting factor for the radial stress at the cylinder-disc interface in terms of blade load. For this calculation, rotational speed is set to zero as it does not affect the factor. Displacement of the cylinder at the interface is calculated using equation (4) for displacement in a long cylinder with outer pressure P.

$$u = \frac{r_i^2 r}{\mathrm{E}} \left\{ \frac{r^2(1-\nu) + r_b^2(1+\nu)}{r^2 \left(r_i^2 - r_b^2\right)} \right\} \cdot P \tag{4}$$

Equations $(2)-(2c)$, $(3a)$ and $(3b)$ are used to calculate a new blade load, F_2, which is used with the actual blade force, F (known), to calculate a correcting factor for radial stress P, see equation (5).

$$P = \frac{F - F_1}{F_2} \tag{5}$$

The displacement at the cylinder is then calculated for the case with rotation and the value of P calculated in (5) by adding the outputs of equations (1) and (4). Equations $(2)-(2c)$, $(3a)$ and $(3b)$ are used to calculate a third blade load, F_3, which is used to find P using (6).

$$P = P + \frac{F - F_3}{F_2} \tag{6}$$

Displacements are recalculated for the case with rotation and the value of P found in (6). This process is iterated, recalculating F_3 and P until the value $F - F_3$ falls within tolerance. Now that the correct value of P has been found, it can be used in conjunction with equations $(7)-(9)$ to calculate stresses in the cylinder.

$$(\sigma_r)_{cyl} = \frac{\rho\omega^2}{8} \cdot \frac{3-2\nu}{1-\nu} \left(r_b^2 + r_i^2 - \frac{r_b^2 r_i^2}{r^2} - r^2 \right) + \frac{r_i^2}{r_i^2 - r_b^2} \left(1 - \frac{r_b^2}{r^2} \right) \cdot P \tag{7}$$

$$(\sigma_t)_{cyl} = \frac{\rho\omega^2}{8} \left\{ \frac{3-2\nu}{1-\nu} \left(r_b^2 + r_i^2 + \frac{r_b^2 r_i^2}{r^2} \right) - \frac{1+2\nu}{1-\nu} \cdot r^2 \right\} + \frac{r_i^2}{r_i^2 - r_b^2} \left(1 + \frac{r_b^2}{r^2} \right) \cdot P \tag{8}$$

$$(\sigma_z)_{cyl} = \frac{\rho\omega^2}{4}\cdot\frac{\nu}{1-\nu}\left(r_b^2 + r_i^2 - 2r^2\right) \qquad (9)$$

Displacement in the cylinder is calculated using the constitutive law (10).

$$u_{cyl} = \frac{r}{\mathrm{E}}\left\{\sigma_r - \nu\left(\sigma_r + \sigma_z\right)\right\} \qquad (10)$$

From the values of u calculated in the last iteration of (2), calculate stresses at each point in the disc using (11) and (12).

$$\varepsilon_r = \frac{du}{dr} \qquad \varepsilon_t = \frac{u}{r} \qquad (11)$$

$$(\sigma_r)_{disc} = \frac{\mathrm{E}}{1-\nu^2}\left[\frac{du}{dr} + \nu\frac{u}{r}\right] \qquad (12a)$$

$$(\sigma_t)_{disc} = \frac{\mathrm{E}}{1-\nu^2}\left[\nu\frac{du}{dr} + \frac{u}{r}\right] \qquad (12b)$$

3. Results and Discussion

The stress and displacement plots for a rotor with the dimensions from Table 1 are shown in Figure 2. Results from this first stage are currently being verified.

Examining the stresses, the radial stresses are of the order 10^1 MPa and the plot exhibits a discontinuity at the cylinder-disc interface due to assumption that the force from the disc is spread evenly over the length of the cylinder stage. Tangential stress in the disc are of order 10^1 MPa, however those in the cylinder are purported to be of order 10^2 MPa and jump by a factor of 10 at the cylinder-disc interface. The calculated displacements are currently erroneous as radial displacement must clearly be continuous from cylinder to disc.

4. Conclusions and Future Work

As the preliminary results are not yet verified, no conclusions can be made about the stresses within the rotor. Work is currently being performed to verify the model outputs from centrifugal loading in two ways. An analytical solution is being found for a simplified geometry consisting of a disc with constant thickness. The model will be run with this geometry and the results will be compared. Also, Verve has supplied results from another program, which was developed to model rotor creep at high temperatures and perform analysis on stress from centrifugal and thermal loading.

Following verification of the stresses and displacements due to body forces, the program will be modified to include the thermal stress component and finally stresses from the shrink fit. These sections will subsequently be verified.

Another element of the program which needs to be further developed is the way which rotor geometry is input. Currently, only linear and hyperbolic disc geometries can be input.

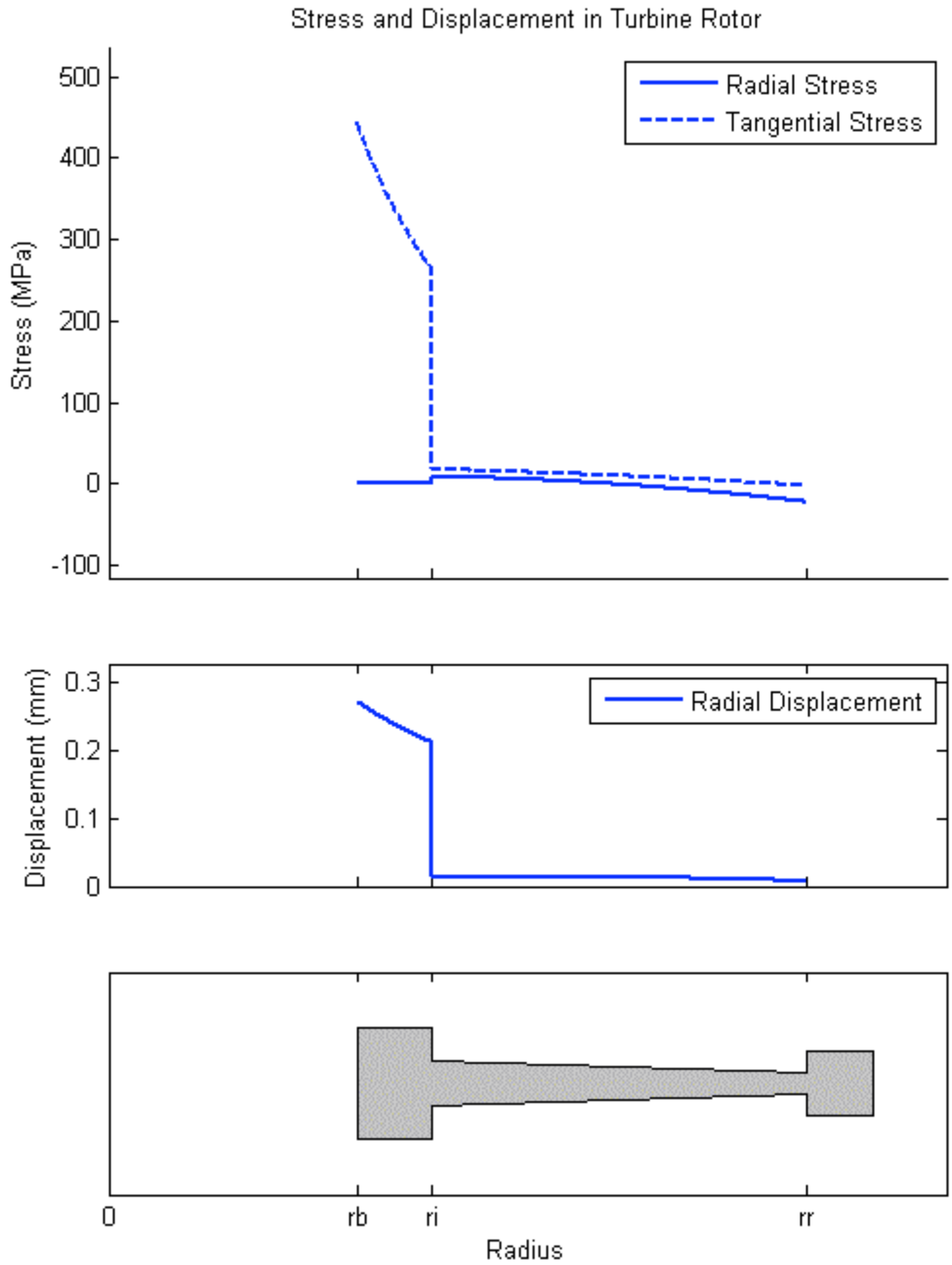

Figure 2 **Stress and discplacement in turbine rotor due to centrifugal loading and blade pressure loading.**

5. References

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